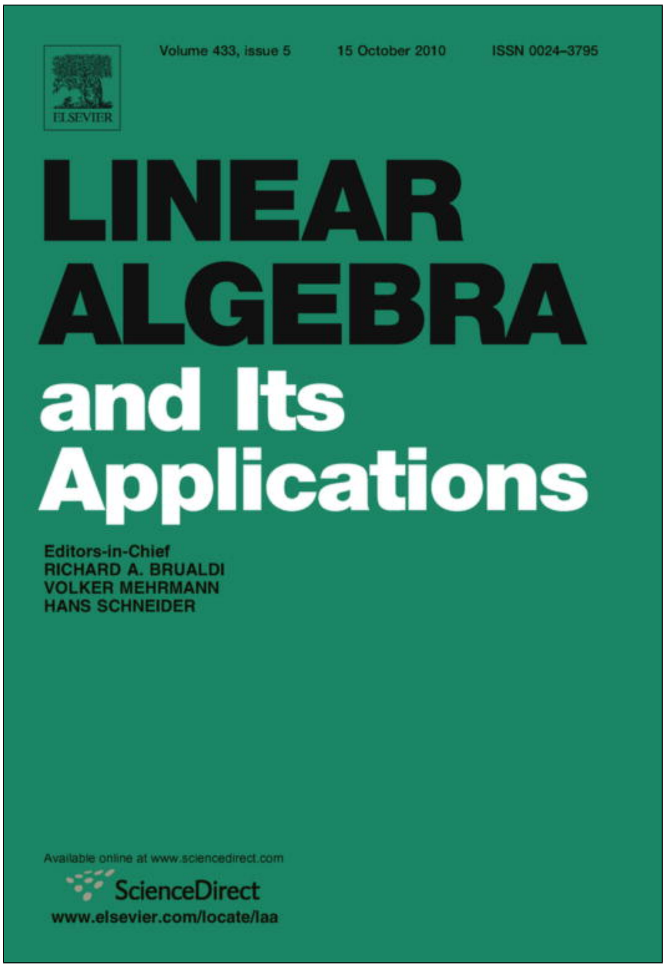
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Book Review

Review of Totally Positive Matrices by Allan Pinkus, Cambridge Tracts in Mathematics, Vol. 181. Cambridge University Press, Cambridge, UK (2010). xi+182 pp., Hardback, ISBN: 978-0-521-19408-2

This monograph is intended as a presentation of the central properties of (strictly) totally positive matrices, the class of matrices which have all their minors nonnegative (respectively, positive).[1] As such it has three predecessors: the books by Gantmacher and Krein [2] and Karlin [3] which both appeared in the sixties as well as the short monograph by Ando [1]. The article by Ando was written in 1984 or earlier and a considerable amount of research has been done since then. So an update and extension of Ando's monograph is certainly warranted.

The book has six chapters. Each starts with a short introduction to the theme and ends with remarks. Here, the presented results are put in historical perspective and references as well as additional material are given.

In the first chapter, basic notation and definitions and various facts and formulae used in the subsequent chapters are introduced. Some operations are presented for which (strict) total positivity of matrices is preserved. Then, as a central property of the totally positive matrices, it is shown that vanishing entries and, more generally, zero values of minors of such matrices are not arbitrary in nature. So, under suitable assumptions of linear independence, such a zero value 'throws a shadow', i. e., all minors of the same order to the left and below it or to the right and above it are zero, too. Based on recent results of the author, it is shown that for a non-singular totally positive matrix there are certain basic vanishing minors formed from consecutive rows and columns from which all other zero minors can be derived. Finally, it is proven that the generalized Hadamard inequality holds for totally positive matrices and some inequalities are deduced from it.

In the second chapter, some determinantal criteria for a matrix to be (strictly) totally positive and results on *LDU* factorizations are presented. Also, a result due to A. Whitney is proven, viz. that the set of the strictly totally positive matrices is dense in the set of the totally positive matrices; by this fact many properties shown to be true for the set of the strictly totally positive matrices are extended (often in a weaker form) to the set of the totally positive matrices. Finally, a different and simple determinantal criterion for strict total positivity, which is based only on the magnitude of the 2-by-2 minors formed from consecutive rows and columns of an entry-wise positive matrix, is presented together with its long proof (taken from the original work by Katkova and Vishnyakova [4]).

The third chapter is concerned with variation diminution. In the fourth chapter various examples of totally positive matrices are listed. For more general classes of matrices like Green, Jacobi, Hankel, Toeplitz, and generalized Hurwitz matrices, conditions are given under which total positivity is obtained. Also, the Hadamard product of two totally positive matrices is considered therein. Whereas the Hadamard product of two totally positive matrices need not be totally positive, there are some classes of totally positive matrices which are closed under the Hadamard product.

[1] Many authors use the terms *totally nonnegative* and *totally positive* for totally positive and strictly totally positive, respectively.

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Chapter 5 reviews the spectral properties of totally positive matrices. The main part of the results are due to Gantmacher and Krein. To keep the exposition self-contained, classic theorems like Perron's and Kronecker's Theorems are proven, too. Finally, in the last chapter various factorizations of (strictly) totally positive matrices are considered.

The book is concluded with an afterword which contains short bibliographical sketches of the four persons who have made outstanding contributions to total positivity, viz. I.J. Schoenberg, M.G. Krein, F.R. Gantmacher, and S. Karlin.

The choice of the topics seems to reflect the author's own research. However, the book under review covers the main results on total positivity; it is self-contained and as such it could serve not only as a reference book but as a classroom text, too. The main results are always put in a historical perspective. For nearly all statements full proofs are given (only in the chapter on the examples and the Hadamard product some proofs are omitted). Furthermore, to some proofs alternatives are provided which are often elaborated by the author. E. g., for Theorem 5.3 in Section 5.2 on spectral properties of oscillation matrices (a class intermediate between the totally positive matrices and strictly totally positive ones) two proofs are presented: The classic one due to Gantmacher and Krein is based on the theorems of Perron and Kronecker. The second proof makes use of variation diminishing and is due to U. Elias and the author.

Many topics are not included in this book, e.g., completion problems, majorization, and accurate computations with totally positive matrices. But with an attempt to make the monograph all-encompassing, certainly the self-containedness would have suffered. The reader may consult the remarks to the chapters for further reading.

On the whole, the author has done an excellent job of writing a well-organized survey which may be of use for anyone who is working in the fields in which total positivity arises, like matrix analysis, combinatorics, approximation theory, computer aided geometric design, and statistics.

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