Session 6

Exercise 1: Single and Double Slit Diffraction formulas

The formulas $d \sin\theta = m\lambda$ for Double Slit Diffraction and $a \sin\theta = m\lambda$ for Single Slit Diffraction are mixed from time to time. Explain the configurations of the underlying experiment, the parameters of the two equations and the meaning of the two equations.

Hint: Refer to the script of module 6, pages 43 to 55.

Exercise 2: Single slit diffraction

Light of a wavelength of 600 nm traverses a single slit. Derive the angle between the optical axis, centered to the slit, and the first diffraction minimum for a slit diameter d of

a) 1 mm

b) 0.1 mm

c) 0.01 mm

Hint: Refer to the script of module 6, pages 43 to 55.

Exercise 3: Double-slit diffraction

The light of a Helium-Neon laser with a wavelength $\lambda = 633$ nm passes a double-slit at vertical incidence. The interference pattern is observed at a distance of 12 m. The distance of the first order to the center maximum is 82 cm.

a) What is the distance d of the two slits?

b) How many maxima are observable?

Hint: Refer to the script of module 6, pages 43 to 55.

Exercise 4: Fraunhofer diffraction problem: Single Slit diffraction

Let an aperture with a diameter a be positioned in the x-axis and illuminated by a plane wave emanating from the aperture $E(x; 0) = 1$.

a) For a Fraunhofer diffraction problem, formulate the problem in the space domain using the Fraunhofer Diffraction Integral

b) Derive the diffracted field in the frequency domain from a)

Hints:

Refer to the script of module 6, pages 56 to 67.

Use the Fraunhofer Diffraction Integral and then substitute $f_x = x'/(\lambda z)$ by considering the substitution rule $f_x(x')/dx\ dx = df_x$.

Drop all pure phase factors before the Fraunhofer Diffraction Integral with the argument that the shifts under Fraunhofer conditions (far-field) are too small to make a difference.

The argument does not hold for the factor $1/\lambda/z$, but the factor vanishes due to the substitution $f_x = x'/(\lambda z)$.

Exercise 5: MATLAB exercise: Spectral analysis of a laser beam with Gaussian beam profile

Given a laser beam with a Gaussian amplitude profile

$$E(x, y; z) = \frac{E_0}{z_R} \frac{W_0}{W(z)} e^{-\frac{x^2+y^2}{W^2(z)}} \cdot e^{j\left(kz - \arctan\left(\frac{z}{zR}\right) + k\frac{x^2+y^2}{2R(z)}\right)}$$

where W_0 is the minimum beam diameter (waist), $z_R = \pi W_0^2/\lambda$ is the Rayleigh range (area of the diverging beam has doubled), $W(z) = (1 + (z/z_R)^2)^{0.5}$ is the beam width at distance z, $R(z) = z + (z_R^2/z)$ is the radius of the beam profile at distance z. $exp(-\arctan(z/zR))$ is called the Gouy phase.

a) Plot the absolute of a Gaussian beam with $\lambda = 850\ nm$ and $W_0 = 50\mu m$ at the distances $0 \cdot zR$, $0.5 \cdot zR$, $1 \cdot zR$ and $2 \cdot zR$ in an aperture $X = 4\mu m$ and a resolution of $8bits$ in the one-dimensional case.

b) Extent the solution in a) to the two-dimensional case. Make the resolution and aperture configurable. Plot the three-dimensional Gaussian for the same distances as in a) and show in a contour plot that the beam opens due to divergence.

c) Calculate the spectrum the Gaussian at the distances in a). Plot the absolute of the spectrum and show that it diverges as the beam diverges.

d) Retrieve the Gaussian field distribution from a superposition of plane wave components, obtained from the spectrum. Perform a step-by-step plot to show how the superposition of plane waves approximates the original Gaussian distribution.

e) The program shall return the maximum signal frequency to avoid aliasing.

Hints: Use a spatial indexing `ix=-nx/2:1:nx/2-1` where `nx=pow2(n)` is a two to the power of `n`. Do equivalent for `iy`.

a) Use the matrix processing of Matlab and the functions `plot` to plot the results

b) Use the functions `meshgrid()`, `surf()`, `contour3()` and `view(2)`

c) Use the functions `fft2()` and `fftshift()`.

d) Spatial frequencies are defined $f = \sin\theta/\lambda$, where $k = 2\pi f$ and θ is the angle of incidence. In the discrete Fourier theory, the i-th frequencies is $f_{ix} = i/X$, derived from the period $X = n_x \triangle x$. n_x is the number of samples. Use the `pause(t)` command

e) What tells the sampling theorem?